Technical Comments

Comment on "Experimental Verification of St. Venant's Principle in a Sandwich Beam"

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THE last paragraph of author's Note[1] motivated the discusser to make some specific comments on the subject, with particular reference to photoelastic model analysis of multiply connected nonhomogeneous bodies and Araldite resins. It is hoped that the comments may be beneficial to the author in planning his future experimental work, which indeed will be very interesting and useful.

The U.S. designation of Araldit D (epoxy resin) is Araldite 502 and is a cold setting resin when used with hardener 951. The discusser is one of the earliest to advocate the use of Araldite 502 in photoelastic investigations[2] in view of its unique properties such as absence of prominent creep and rind effect, cold setting, the resin can be cast on metallic components (no shrinkage or temperature stresses), valuable in nonhomogeneous structures, powerful adhesive action, etc. In recent years a large number of prominent photoelasticians used Araldite 502 as model material for special projects.

The Young's modulus of the resin[2] prepared with 10% hardener 951 is 3.75×10^5 psi while the addition of plasticizers such as cyclohexanol, dibutyl phthalate, and others can substantially decrease the value of Young's modulus[3] of the cured resin to a value of about 500 psi and thus make the core material photoelastically sensitive.

The second part of the comment deals with multiply connected bodies loaded by concentrated forces which do not reduce to a zero resultant force or a couple on any boundary. If the concentrated load in Fig. 1 of Ref. 1 is applied at B, the stress field in the sandwich beam is a function of Poisson's ratios of the core and face materials. In multiply connected homogeneous bodies, the stress distribution as predicted by photoelastic techniques, is generally slightly different from that in the prototype,[4] and Bickley[5] estimates the order of magnitude as 7%. In multiply connected nonhomogeneous bodies such as sandwich beams, the Poisson's ratios of face and core material, respectively, in case 1 of Ref. 1 may be assumed to be of the order of 0.49 and 0.3, and the error estimates in such situations are not clearly defined, as far as photoelastic analysis goes. Therefore caution should be exercised in interpreting the results of photoelastic model experiments to duplicate nonhomogeneous multiply connected bodies. It may be interesting to extend the work to the case where the load point B is located in the bottom surface of the beam, but still statically equivalent to load system A; or a dislocation may be introduced at the present load point B so as to annihilate the dependence of the stresses on Poisson's ratios. In either case, the sandwich beam is a simply connected nonhomogeneous elastic plane body; and the problem can be handled by using conventional photoelastic techniques.

References

[1] Alwar, R. S., "Experimental Verification of St. Venant's Principle in a Sandwich Beam," *AIAA Journal*, Vol. 8, No. 1, Jan. 1970, pp. 160–162.

[2] Amba-Rao, C. L., "The Suitability of Araldite D Resin in Photoelastic Investigations," *British Journal of Applied Physics*, Vol. 7, No. 6, June 1956, p. 229.

[3] Dally, J. W., Durelli, A. J., and Riley, W. F., "A New Method to 'Lock-in' Elastic Effects for Experimental Stress Analysis," *Journal of Applied Mechanics*, Vol. 25, No. 2, June 1958, pp. 189–195.

[4] Frocht, M. M., *Photoelasticity Vol. II*, Wiley, New York, 1948, pp. 168–193.

[5] Bickley, W. G., "The Distribution of Stress Round a Circular Hole in a Plate," *Philosophical Transactions of the Royal Society of London*, Vol. 227, 1928, pp. 383–415.

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Reply by Author to C. L. Amba-Rao

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THE author is thankful to Amba-Rao for suggesting some materials that can be used for the core.

It is true that in multiply connected bodies with nonzero stress resultants, the stress distribution is dependent on Poisson's ratio, and one has to be careful in extrapolating the results to a prototype with entirely different materials. The object of the last paragraph in my paper is to indicate that the $E_{\text{face}}/E_{\text{core}}$ values considered in the experiments are not far from the values which occur in practice.

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Comment on "Generalized Stiffness Matrix of a Curved-Beam Element"

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ALTHOUGH the author has clearly developed the stiffness matrix for a curved beam element,[1] the existence of three previous papers on the subject should be noted. In 1965 Tezcan and Ovunc[2] presented the stiffness matrix relative to the radial, tangential, and transverse axes of a curved member arbitrarily oriented in space, as well as the special cases of plane frame and grid members. This work was reproduced

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independently by Morris[3] who derived the stiffness matrix relative to an orthogonal coordinate system common to both ends of the member. For a study of curved-girder bridges, Sawko[4] derived the flexibility matrix for a curved beam element, but did not give an explicit form for the stiffness matrix. Numerical studies were discussed in all three papers.

References

[1] Lee, H.-P., "Generalized Stiffness Matrix of a Curved-Beam Element," *AIAA Journal*, Vol. 7, No. 10, Oct. 1969, pp. 2043–2045.

[2] Tezcan, S. and Ovunc, B., "Analysis of Plane and Space Frameworks with Curved Members," *Publications of the International Association for Bridge and Structural Engineering*, 1965, pp. 339–352.

[3] Morris, D. L., "Curved Beam Stiffness Coefficients," *Journal of the Structural Division*, ASCE, Vol. 94, No. ST5, May 1968, pp. 1165–1174.

[4] Sawko, F., "Computer Analysis of Grillages Curved in Plan," *Publications of the International Association for Bridge and Structural Engineering*, 1967, pp. 151–170.

Comments on "Hypersonic Flow Past an Unyawed Cone" and "Shoulder Pressures for Slender Cone-Afterbody Combinations in Hypersonic Flow"

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In Ref. 1, Rasmussen has developed an interesting procedure for calculating the hypersonic flow past an unyawed cone. In this procedure, the stream function differential equation in the hypersonic small-disturbance limit is recast as an integral equation, which is then solved by successive approximations. The approximate solution resulting from this analysis has subsequently been used in Ref. 2 to calculate the shoulder pressure on a cone-afterbody configuration.

In Ref. 1, the shock angle is computed from the zeroth-order approximation for the stream function. The surface pressure, on the other hand, is derived from the first-order approximation for the stream function but, in the process, the zeroth-order solution for the shock angle is incorporated. Thus, it seems that a certain degree of arbitrariness is introduced. Nevertheless, the proposed formulas have the advantage of being simple as well as accurate (provided that the hypersonic similarity parameter K is not too small). Higher-order approximations presumably would improve the accuracy of the solutions for small values of K (when the shock does not lie so close to the body) but such calculations would run into severe algebraic complications.

Now, it is interesting to observe that the proposed formulas for shock angle and surface pressure actually are constant-density solutions. In fact, the hypersonic small-disturbance equations for plane or axi-symmetric flow admit of constant-density solutions in closed explicit form for arbitrary body shape. The simplified equations are

$$(\partial/\partial y)[vy^j] = 0, \ \partial v/\partial x + v\partial v/\partial y = -\tfrac{1}{2}\epsilon\partial C_p/\partial y$$

with shock conditions $v_s = (1-\epsilon)G'$, $C_{p_s} = 2(1-\epsilon)G'^2$ and surface tangency condition $v_b = F'$. Conventional notation has been used. $F(x)$ and $G(x)$ represent the body shape and shock shape; ϵ is the preshock-postshock density ratio; v is nondimensionalized by u_∞. The foregoing system is readily solved, the results being

$$v = \begin{cases} F' & (j=0) \\ FF'/y & (j=1) \end{cases}$$

$$G = \begin{cases} F/(1-\epsilon) & (j=0) \\ F/(1-\epsilon)^{1/2} & (j=1) \end{cases}$$

$$C_p = \begin{cases} [2/(1-\epsilon)](F'^2 + FF'') - (2/\epsilon)(y-F)F'' & (j=0) \\ \dfrac{1}{\epsilon}(F'^2 + FF'')\ln\left[\dfrac{F^2}{(1-\epsilon)y^2}\right] + F'^2 + \dfrac{1}{\epsilon}\times \\ \qquad (1 - F^2/y^2)F'^2 & (j=1) \end{cases}$$

For conical flow, we find,

$$\frac{C_p}{\theta_b^2} = 1 + \frac{1}{\epsilon}\left[\ln\left(\frac{\theta_b}{\theta}\right) - \left(\frac{\theta_b}{\theta}\right)^2 + \ln(1-\epsilon)^{-1} + 1\right]$$

where we have replaced y by θx.

Specializing to a perfect gas with constant specific heats and adiabatic index γ, we further find,

$$\epsilon = [(\gamma-1)K^2 + 2]/[(\gamma+1)K^2 + 2]$$

where $K = M_\infty\theta_b$. Substitution into the solutions for shock angle and pressure coefficient yields results identical with those obtained by Rasmussen's procedure.

References

[1] Rasmussen, M. L., "On Hypersonic Flow Past an Unyawed Cone," *AIAA Journal*, Vol. 5, No. 8, Aug. 1967, pp. 1495–1497.

[2] Fiorino, T. D. and Rasmussen, M. L., "Shoulder Pressures for Slender Cone-Afterbody Combinations in Hypersonic Flow," *AIAA Journal*, Vol. 7, No. 1, Jan. 1969, pp. 169–170.

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